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Knowledge Representation meets DataBases

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KRDB Research Centre Technical Report:

Controlled English for DL-Lite

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Abstract

As it is well-known, querying and managing structured data in natural language is a challenging task due to its ambiguity (syntactic and semantic) and its expressiveness. On the other hand, querying, for example, a database is a well-defined and unambigous task, namely, that of evaluating some formal query (e.g. an SQL query) of a limited expressiveness over some finite structure: the instance of the database schema. However these formal query languages may be utterly obscure for the casual user. To bridge this gap, the use of controlled languages has been proposed. Controlled languages are fragments of, say, English, with a limited vocabulary and a very restricted set of grammar rules, but in which ambiguity is minimal if not alltogether unexistent. Moreover, they can be engineered in a way that a meaning representation built out from some logic can be compositionally constructed during parsing. A logic ideally matching the expressive power of a formal query language being able to be taken as such. As a first step to building such a language we study the crucial issue of the expressive power of the logic to which it should translate in the back end: DL-Lite, that we believe well-adapted for querying and specifying data. To this end, we inspect a certain number of tractable fragments of English that compostionally translate into fragments of first order logic (FOL), tractable in that the satisfiability problem (SAT) for these induced fragments of FOL is in **P**, and compare them to DL-Lite w.r.t. expressive power.

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Introduction

Data is ubiquitous. Whether stored in databases or in knowledge bases the task of structuring, modelling, declaring, updating and querying it, is not all trivial. Database management systems (DBMS) like, say, dBASE or Oracle, are an attempt to carry on with these tasks. The interfaces of these systems are based in formal query languages that combine both declarative and imperative features such as SQL or (full) Datalog – whose expressive power is equal to that of first order logic (FOL) for the former and strictly greater for the latter (cf. [11]).

But using these languages requires some previous training and can prove counterintuitive to the a casual end user. For such a user the intuitive appeal and understanding of the machine interface can be crucial. It would thus be suitable in such cases to shift to natural language (NL) and to use natural language questions instead of formal queries, an approach that has been widely studied among the natural language interfaces to databases (NLIDB) community (cf. [14, 13, 1]). NL, however, is overridden with ambiguity, whether lexical, structural or semantical (cf. [12]). Retrieving data from a DB by means of NL questions can exceed by far these tight expressivity and complexity bounds on QA. We therefore believe that to address this problem a compromise between expressive power and the intuitive appeal of NL has to be reached and will argue further that this compromise involves the use of the so-called *controlled languages* (CLs), which are fragments of NL tailored to deal with these tasks and where utterances compositionally translate (*modulo* some compostional translation ϕ) into a logical expression called *meaning representation* (MR), that encodes semantics at the sentence level (cf. [18]).

But then, the expressive power of the MR formalism, as well as its relationship (*modulo* the compositional translation ϕ) with NL, is of utmost importance. This report aims at providing some light into this latter issue. We thus formally study a knowledge representation language, DL-Lite, particularly well-suited for data management (cf. [7, 6]), and compare it in expressive power to two fragments of English, COP and COP+TV+DTV, or at any rate with the fragments of first order logic (FOL) they induce by compositionally translating into them as MRs.

The structure of this report is as follows. The first section recalls the definition of DL-Lite, both at the level of its syntax and of its semantics. The second will introduce the fragments of English COP and COP+DTV+TV. The third will provide a definition of expressive power for logics in general and state a number of results regarding DL-Lite and FOL. The fourth section will compare the expressive power of DL-Lite to that of the aforementioned fragments of English. Finally, the last section will be devoted to the concluding remarks.

Chapter 1

DL-Lite

We begin by recalling the definition of DL-Lite's syntax and semantics, as stated by Calvanese *et al.* in [7, 6]. See also [2] for further details on description logics.

Definition 1.0.1 *Let $\mathcal{P} = \{P_i | i \in \mathbb{N}\}$ and $\mathcal{R} = \{R_i | i \in \mathbb{N}\}$ be two countable sets of primitive concept and role symbols. Right hand side concepts occur at the right hand side of the inheritance or inclusion relation symbol $\sqsubseteq$. Left hand side concepts, occur to its left. DL-Lite* left hand side concepts C *and* right hand side concepts $\mathcal{B}$ *are defined as follows:*

1. $\mathcal{B} ::= \mathcal{P} \mid \exists\mathcal{R} \mid \exists\mathcal{R}^- \mid \mathcal{B} \sqcap \mathcal{B}$.

2. $C ::= \neg\mathcal{P} \mid \neg\exists\mathcal{R} \mid \neg\exists\mathcal{R}^- \mid \mathcal{B} \mid C \sqcap C \mid \exists\mathcal{R}\colon C \mid \exists\mathcal{R}^- : C$.

Thinking in terms of the well-known ER diagram formalism, concepts can be styled formal counterparts of entities (or classes), representing collections of individuals, and roles, as binary associations linking entities and thus holding over the individuals belonging to the class. Next, assertions:

Definition 1.0.2 *Let $\mathcal{K} = \{c_i | i \in \mathbb{N}\}$ be a set of constants. DL-Lite* A-Box assertions $\mathcal{A}$ *and* A-Box assertions $\mathcal{T}$ *are defined as follows:*

1. $\mathcal{A} ::= \mathcal{B}(\mathcal{K}) \mid \mathcal{R}(\mathcal{K}, \mathcal{K}) \mid C(\mathcal{K})$

2. $\mathcal{T} ::= \mathcal{R} \sqsubseteq \mathcal{R} \mid \mathcal{B} \sqsubseteq C$.

So, right hand side concepts occur at the right hand side of the inheritance or inclusion relation symbol $\sqsubseteq$. Left hand side concepts, occur to its left. For instance, in Figure 1.1 below, *Man* is a left hand side concept and $\exists$*Loves* a right hand side concept for the assertion *Man* $\sqsubseteq$ $\exists$*Loves*. Making this distinction is important because concept contructors do not apply irrestrictedly to previously defined concepts, but according to wether they figure to the right or the left of the inheritance symbol. For example, the concept $\exists$*Loves*: *Woman* can only occur to the right, thus precluding an assertion like $\exists$*Loves*: *Woman* $\sqsubseteq$ *Man* (i.e the converse of the previous example) – they can only be stated as necessary, but not sufficient, conditions of the concept occuring to the left. Assertions containing constants are called *facts* otherwise they are called *terminological* assertions.

Definition 1.0.3 (Knowledge Bases) *A DL-Lite* knowledge base *(KB) is a tuple* $KB = \langle ABox, TBox \rangle$*, where the ABox is a set of facts and the TBox a set of terminological assertions.*

The ABox is also known as the *extensional* knowledge base and the TBox as the intensional *knowledge base*. And now we turn to descriptive semantics. Descriptive semantics verifies the unique name assumption (UNA), namely that any two pairwise distinct individual constants must be mapped to pairwise distinct elements of the interpretation domain.

Definition 1.0.4 *A descriptive semantics* interpretation *is a tuple* $I = \langle \Delta^{\mathfrak{I}}; .^{\mathfrak{I}} \rangle$ *where:*

1. $\Delta^{\mathfrak{I}}$ *is a non empty possibly countably infinite set called the* domain.
2. $.^{\mathfrak{I}}$ *is an interpretation function defined over concepts (both left and right hand side) and roles as follows:*
   (a) $c^{\mathfrak{I}} \in \Delta^{\mathfrak{I}}$*, for every constant c. Furthermore,* $.^{\mathfrak{I}}$ *is injective on constants.*
   (b) $P^{\mathfrak{I}} \subseteq \Delta^{\mathfrak{I}}$*, for every basic concept symbol P.*
   (c) $R^{\mathfrak{I}} \subseteq \Delta^{\mathfrak{I}} \times \Delta^{\mathfrak{I}}$*, for every role symbol R.*
   (d) $(\exists R)^{\mathfrak{I}} = \{x \in \Delta^{\mathfrak{I}} | \exists y \in \Delta^{\mathfrak{I}} \text{ s.t. } \langle x, y \rangle \in R^{\mathfrak{I}}\}$
   (e) $(\exists R^{-})^{\mathfrak{I}} = \{y \in \Delta^{\mathfrak{I}} | \exists x \in \Delta^{\mathfrak{I}} \text{ s.t. } \langle x, y \rangle \in R^{\mathfrak{I}}\}$
   (f) $(B \sqcap B')^{\mathfrak{I}} = B^{\mathfrak{I}} \cap B'^{\mathfrak{I}}$.
   (g) $(\neg B)^{\mathfrak{I}} = \Delta^{\mathfrak{I}} - B^{\mathfrak{I}}$.
   (h) $(\exists R : C)^{\mathfrak{I}} = \{x \in \Delta | \exists y \in \Delta^{\mathfrak{I}} \text{ s.t. } y \in C^{\mathfrak{I}} \text{ and } \langle x, y \rangle \in R^{\mathfrak{I}}\}$.
   (i) $(\exists R^{-} : C)^{\mathfrak{I}} = \{y \in \Delta^{\mathfrak{I}} | \exists x \in \Delta^{\mathfrak{I}} \text{ s.t. } x \in C^{\mathfrak{I}} \text{ and } \langle x, y \rangle \in R^{\mathfrak{I}}\}$.
   (j) $(C \sqcap C')^{\mathfrak{I}} = C^{\mathfrak{I}} \cap C'^{\mathfrak{I}}$.

An interpretation $\mathfrak{I}$ is said to be a *model* of a T-Box assertion of the form $B \sqsubseteq C$, denoted $\mathfrak{I} \models_{ds} B \sqsubseteq C$ iff $B^{\mathfrak{I}} \subseteq C^{I}$. It is said to be a *model* of an A-Box assertion $C(a)$, denoted $\mathfrak{I} \models_{ds} C(a)$ iff $a^{\mathfrak{I}} \in C^{\mathfrak{I}}$. And analogously for role subsumption assertions. It is finally said to be a *model* of a knowledge base KB, denoted, again, $\mathfrak{I} \models_{ds} KB$, iff it is a model of all the assertions of its A-Box and its T-Box. A concept C is said to be *satisfiable* iff there exists an interpretation $\mathfrak{I}$ such that $C^{\mathfrak{I}} \neq \emptyset$. A T-Box (resp. an A-Box) is said to be *satisfiable* iff its has a model. Similarly, a knowledge base KB is said to be *satisfiable* exactly when there is an interpreation $\mathfrak{I}$ that is both a model of its A-Box and of its T-Box. Finally, a knowledge base KB is said to *entail* an assertion α, denoted $KB \models_{\text{DL-Lite}} \alpha$, iff every model of KB is a model of A.

Remark 1.0.1 We observe that:

- The satisfiablity problem for DL-Lite is in **P** (cf.[7, 6]).
- DL-Lite is the maximal tractable description logic capable
- It can thus be used as a conceptual modelling formalism with the advantage that one can perform efficient automated reasoning on it – indeed reasoning services (like QONTO cf. [7]) can be tuned to it to attain this aim.

T-Box

$MarriedMan \sqsubseteq Man$
$Man \sqsubseteq \exists Loves$
$Man \sqsubseteq \exists Loves\colon Woman$
$Man \sqsubseteq LivingCreature$

A-Box

$Man(James)$
$MarriedMan(John)$
$Loves(John, Mary)$
$(\exists Loves\colon Woman)(John)$

Figure 1.1: The DL-Lite knowlege base KB_s

- DL-Lite is FOL-reducible. This means that query answering and satisfiability is **LOGSPACE** in data complexity – i.e. on the size of the ABox, in other words, in the number of constants occurring within the ABox (cf.[7, 6]).

As it is well-known, DL-lite, like other description logics, correspond to a fragment of FOL, albeit with a somewhat different notation. It is then routine to translate, by means of a model-preserving translation into a fragment of first order logic (FOL). We shall see how this translation works in more detail in the next section.

Figure 1.1 shows a very simple example of a knowledge base partially describing, one would say, the domain of men. Just for illustration, we can see that following the definitions above we have that $KB_s \models_{\text{DL-Lite}} Man(John)$.

Chapter 2

COP and COP+TV+DTV

In this section we give a very brief overview of Pratt's controlled fragments of English (cf.[15, 16, 17]), namely COP and COP+TV+DTV. They are fragments defined to capture very simple syntactic and semantic phenomena of English – basically, very simple inference patterns, such as the syllogism in the case of COP. They are defined via phrase structure granmmars augmented with a semantic mapping that allows for building a semantic representation incrementally following the parse tree of each phrase or sentence: as it stands a first order logic (FOL) sentence or closed formula built by beta reduction and higher order logic (HOL). Pratt defines the expressive power of these fragments as that of the fragments of FOL associated to them by the semantic mapping – i.e. as that of the set of their meaning representations. Because of this, in what follows we will not distinguish among the fragments and their logics. As Pratt's grammars contain no recursive rules, the number of sentences generated and the size of the sets of meaning representations depends on the number of function and content words of our grammar – usually finite and that entails that the former are usually finite as well.

It is interesting to remark that in his paper, he proves that these are actually the only tractable fragments of English w.r.t. satisfiability. The satisfiability problem for these two fragments is decidable and is in **P**. But as soon as we add (cf. [15, 16, 17]) rules dealing with the relative clause (which, by and by, turn the grammar recursive and the fragment infinite), we lose tractability. For instance COP+REL (i.e. COP with relative clauses), is already **NP-Complete** (by reduction of 3-SAT). Restricted anaphora turns it into an **EXPTIME-Complete** problem and unrestricted anaphora turns it altogether undecidable. The fragments are defined as follows:

COP	=	Copula, common and proper nouns, negation, universal and existential quantifiers.
TV	=	Transitive verbs (e.g. ”reads”).
DTV	=	Distransitive verbs (e.g., ”gives”).
REL	=	Relative pronouns (i.e., ”who”, ”that”, ”which”, etc.).
RA	=	Restricted (intrasentential) anaphora.
GA	=	Generalized anaphora.

Actually they can constitute a lattice (ordered by inclusions) of 2^5 possible fragments. But those are the only ones that deserve attention. Table 2.1 below summarizes the complexity results for the satisfiability (SAT) problem of the fragments, that is, of the induced fragments of FOL:

Fragment	**Decision class for satisfiability**
COP	**P**
COP+TV+DTV	**P**
COP+REL	**NP-Complete**
COP+REL+TV	**EXPTIME-Complete**
COP+REL+TV+DTV	**NEXPTIME-Complete**
COP+REL+TV+RA	**NEXPTIME-Complete**
COP+REL+TV+GA	undecidable

Table 2.1: Expressivity of the fragments of English

2.1 COP

COP is the fragment that deals with copula, proper nouns or names, common nouns, adjectives together with the quantifiers "every" and "some", although intransitive verbs can be added to it without any difference in expressive power (if taken to mean something like a copula with a noun). A typical COP sentence is for example:

Every man is mortal.

It is meant to formally capture syllogistic arguments like:

Every philosopher is a man	$\rightsquigarrow$	$\forall x[philosopher(x) \rightarrow man(x)]$
Socrates is a philosopher	$\rightsquigarrow$	$philosopher(Socrates)$
$\therefore$ Socrates is a man	$\rightsquigarrow$	$\therefore man(Socrates)$

It is defined by means of a shrase structure grammar over which a semantic mapping (or compositional translation mapping ϕ, cf. [12]) has been defined. This function builds bottom-up a FOL meaning representation by mapping components to higher order logic (HOL) or type-theoretical expressions (cf. [4, 8, 10]) and then applying beta-reduction on beta-redexes. Table 2.2 below recalls how the fragment is defined and how ϕ is supposed to be compositionally computed on the parse tree:

HOL gives the semantics at the lexical level, and FOL, the semantics at the sentence level, as can be seen in the parse tree below (see Figure 2.1).

2.2 COP+TV+DTV

COP+TV+DTV is the extension of COP that, along with categories and constituents dealt with by COP, allows for phrases built using transitive and distransitive verbs, like "loves" and "gives", respectively. Typical COP+TV+DTV sentences are those of the following forms:

Phrase Structure Rules	MR (= ϕ)
IP → **NP I′**	$(\phi(\mathbf{NP}))\phi(\mathbf{I'}) \rhd_\beta \phi(\mathbf{IP})$
I′ → is a **N**	$\phi(\mathbf{I'}) = \phi(\mathbf{N})$
I′ → is not a **N**	$\phi(\mathbf{I'}) = \neg\phi(\mathbf{N})$
NP → **PropN**	$\phi(\mathbf{NP}) = \phi(\mathbf{PropN})$
NP → **Det N**	$(\phi(\mathbf{Det}))\phi(\mathbf{N}) \rhd_\beta \phi(\mathbf{NP})$

Content lexicon	MR (= ϕ)
N → woman	$\phi(\mathbf{N}) = \lambda x(woman)x$
N → man	$\phi(\mathbf{N}) = \lambda x(man)x$
N → human	$\phi(\mathbf{N}) = \lambda y(human)x$
PropN → Mary	$\phi(\mathbf{PropN}) = \lambda P(P)m$

Function lexicon	MR (= ϕ)
Det → every	$\phi(\mathbf{Det}) = \lambda P\lambda Q\forall x[(P)x \rightarrow (Q)x]$
Det → no	$\phi(\mathbf{Det}) = \lambda P\lambda Q\forall x[(P)x \rightarrow \neg(Q)x]$
Det → some	$\phi(\mathbf{Det}) = \lambda P\lambda Q\exists x[(P)x \wedge (Q)x]$

Table 2.2: COP's phrase structure grammar.

John loves Mary.

Or, for distransitive verbs:

Every catholic gives a dime to a beggar.

Where the number of noun phrase constituents with which they are combined is said to be their arity.

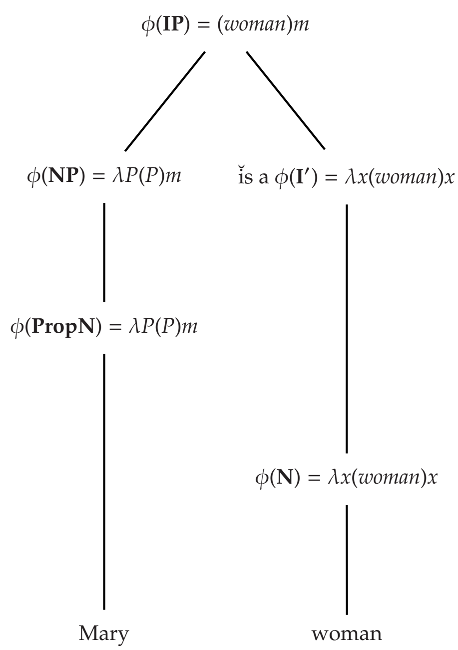

Figure 2.1: Parsing "Mary is a woman" in COP.

Chapter 3

Expressive Power

In this section we study the expressive power w.r.t. to satisfiability of the description logic DL-Lite and two fragments of English (that is, two controlled languages) defined by Pratt in [16, 17], namely COP and COP+TV+DTV. COP is broadly speaking the fragment of English restricted to sentences in present tense and indicative mood containing only proper or common nouns, negation, copula and the quantifiers ”every” and ”some”. The latter fragment is an extension of COP to which rules dealing with transitive (TV) and distransitive verbs (DTV) – verbs of arity, respectively, 2 and 3. Their grammar allows for a compositional account of meaning via a semantic mapping. Hence, to each sentence in either of these controlled languages is associated a meaning representation by way of a first order sentence (closed formula).

Pratt studies the expressive power of these fragments (together with further extensions) relatively to that of the first-order logic fragment that constitutes the (finite) set of their as well as the complexity of the satisfiability problem for these logics, that is in **P** for both COP and COP+TV+DTV. To compare therefore their expressive power with that of DL-Lite, we just compare that of their logics w.r.t. DL-Lite. As we shall see, they are not comparable. However, there are properties that can be expressed at the same time by DL-Lite, COP and COP+TV+DTV. A controlled language – a fragment of English in the vein of Pratt – will be thus tailored in the next section in order to capture this.

Last, but not least, we recall that COP and COP+TV+DTV are the only fragments in Pratt's classification for which satisfiability is tractable. The complexity of the satisfiability problem for other fragments exceeds by far, when decidable, that of these two fragments.

3.1 Expressive Power in General

The expressive power of a logic consists, intuitively, in the properties it can formalize. Hence, in the models it can characterize. But this can be also said otherwise, namely, as the set of logical truths and of entailments it contains – which depend on and are ultimately defined by means of its models (cf.[20, 9], and specifically the chapters on FOL model theory). In the latter sense, the aim pursued is that of reducing the entailment or reasoning problem of one logic to that of the other. Thus, to compare the expressive power of two logics we can choose among two methods: we can either reduce the entailment problem of one to that of the other, or we can reason about the properties the logics are supposed to express. In defining what is a logic and what is its expressive power we follow mainly [19] and [5].

Definition 3.1.1 *Let Σ be some signature. A* logic *is a tuple $\Lambda = \langle \mathfrak{L}_\Lambda, \models_\Lambda \rangle$ where:*

1. *$\mathfrak{L}_\Lambda \subseteq \Sigma^*$ is a* language *over Σ.*
2. *$\models_\Lambda \subseteq \mathfrak{P}(\mathfrak{L}_\Lambda) \times \mathfrak{L}_\Lambda$ is a* consequence *relation (a pre-order) defined by means of a semantics s for $\mathfrak{L}_\Lambda$.*

As usual, elements of $\mathfrak{L}_\Lambda$ are called *formulae*. They are said to be *open* exactly when they contain unbound or free variables, *closed* otherwise. In this latter case they are called *sentences*.

Definition 3.1.2 *Let Λ, Λ' be two logics. Λ' is said to be* at least as expressive as *Λ, denoted $\Lambda \lesssim \Lambda'$, iff there exists a translation $.^*$ from $\mathfrak{L}_\Lambda$ to $\mathfrak{L}_{\Lambda'}$ such that, for any sentence A and any set Γ of sentences of Λ we have that:*

$$(\dagger)\, \Gamma \models_\Lambda A \textit{ iff } \Gamma^* \models_{\Lambda'} A^*$$

The translation $.^*$ is called in such case a *satisfaction-preserving* or a *consequence-preserving* translation, or by others, a *logic homeomorphism* (cf. [19]). If it is injective, a *logic embedding* – and a *logic isomorphism* if it is bijective.

Definition 3.1.3 *Let Λ, Λ' be two logics. Λ is said to be* as expressive as *or* equally expressive as *Λ', denoted $\Lambda \sim \Lambda'$, iff $\Lambda \lesssim \Lambda'$ and $\Lambda' \lesssim \Lambda$.*

Proposition 3.1.1 *We have that:*

1. *$\lesssim$ is a pre-order.*
2. *$\sim$ is an equivalence relation.*

(*Proof*) Indeed:

(1) $\lesssim$ is reflexive and transitive:

- Take for $.^*$ the identity mapping. Then $\Lambda < \Lambda$ for any logic Λ. That is, $<$ is reflexive.
- Let Λ, Λ' and Λ'' be three logics such that $\Lambda \lesssim \Lambda'$ through translation $.^*$ and $\Lambda' \lesssim \Lambda''$ through translation $.^{*'}$ and put $.^{*''} = .^{*'} \circ .^*$ – i.e. take the composition of the two mappings. Then $.^{*''}$ is a mapping from $\mathfrak{L}_\Lambda$ to $\mathfrak{L}_{\Lambda''}$ satisfying $(\dagger)$, whence $\Lambda \lesssim \Lambda''$. In other words, $\lesssim$ is transitive.

(2) By definition, $\sim$ is the symmetric closure of $\lesssim$ and by (1) we know that $<$, hence $\sim$, is reflexive and transitive. Therefore, it is an equivalence relation. □

Proposition 3.1.2 *Let Λ, Λ' be two logics such that $\Lambda \lesssim \Lambda'$. If $.^*$ is bijective, then Λ is equally expressive as Λ'.*

(*Proof*) If $\Lambda \lesssim \Lambda'$, (†) holds from $\mathfrak{L}_\Lambda$ to $\mathfrak{L}_{\Lambda'}$. Now, since $.^*$ is bijective, then $.^{*^{-1}}$ is a bijection from $\mathfrak{L}_{\Lambda'}$ to $\mathfrak{L}_\Lambda$ verifying (†), which proves the result. □

By means of these properties we can formally compare the expressive power of two logics. But there is an alternative way, mainly for negative results where what we look for is to disprove that (†) holds and that the existence of a satisfaction-preserving translation is impossible. A semantics s for a logic Λ serves, among other things, to define the class of its *models*, that is, of the algebraic structures that turn true the sets of sentences from $\mathfrak{L}_\Lambda$. A semantic property is some class of algebraic structure that may or may not coincide with these classes of models. We denote $Mod(\Gamma)$ the class of models of any given set of sentences Γ of some arbitrary logic. If they coincide, it is said to be expressible in this logic. Semantic properties provide a necessary condition for expressibility. Formally:

Definition 3.1.4 *Given a logic Λ, a semantic property $\mathfrak{K}$ is said to be* expressible *in Λ iff there exists a set Γ of sentences of $\mathfrak{L}_\Lambda$ such that $Mod(\Gamma) = \mathfrak{K}$, i.e. its class of models coincides with this property.*

Proposition 3.1.3 *Let Λ, Λ' be two logics s.t. $\Lambda \lesssim \Lambda'$. Then every semantic property expressible in Λ is expressible in Λ'.*

(*Proof*) Suppose the contrary and let $\mathfrak{K}$ be some class expressible in Λ but not in Λ'. Let Γ be the set expressing this property. Then, since (†) holds, there exists a set Γ^* from Λ' such that $\mathfrak{K} = Mod(\Gamma^*)$. Contradiction. □

Hence, in order to determine whether a logic Λ is not as least as expressive as a logic Λ', it is enough to find a property expressible in Λ but not in Λ'. However, this does not preclude their expressive power from overlapping:

Definition 3.1.5 *Let Λ and Λ' be two logics. Λ is said to* overlap *in expressive power with Λ' iff there exists a property (or class of structures) $\mathfrak{K}$ expressible both in Λ and in Λ'.*

3.2 Expressive Power of FOL

In this section we recall briefly FOL semantics and model theory and we introduce a very useful semantic property of FOL, namely that of closure under union of chains. A property that we will use in the following subsections below.

Definition 3.2.1 *Let Σ be a first order signature. A first order* interpretation structure *over Σ is a tuple:*

$$\mathfrak{M} = \langle M; \{R_i^{\mathfrak{M}}\}_{i\in I}; \{f_j^{\mathfrak{M}}\}_{j\in J}; \{c_k^{\mathfrak{M}}\}_{k\in K}\rangle$$

Where:

1. *M is a non-empty set called* domain,

2. *The* $R_i^{\mathfrak{M}}$*'s are n-ary relations over M, for* $n \geq 1$. *They are associated to the n-ary relation symbols of* Σ *of which they constitute the* interpretation.

3. *The* $f_j^{\mathfrak{M}}$*'s are n-ary functions from* M^n *to M, for* $n \geq 1$. *They are associated to the n-ary function symbols of* Σ *of which they constitute the* interpretation.

4. *The* $c_k^{\mathfrak{M}}$*'s are distinguished elements of M. They are associated to the constant symbols of* Σ *of which they constitute the* interpretation.

The index i (resp. j) is called the *position* of the relation $R_i^{\mathfrak{M}}$ (resp. function $f_j^{\mathfrak{M}}$). Note that these families, together with the domain, might be uncountably infinite, although, for the sake of argument, we may assume that they are at most countably infinite. Interpretation structures are thus algebraic structures that act as models of FOL sentences and sets of sentences, by means of an interpretation function that maps constants, relation and function symbols to distinguished elements, relations and functions and that is then extended into an assignment dealing with the free variables of the formulas and a recursively defined evaluation function over these formulae.

Indeed, given an interpretation structure $\mathfrak{M}$, assingments are (partial) functions of the form $v\colon Var(\mathfrak{L}_{\text{FOL}}) \to M$ – where $Var(\mathfrak{L}_{\text{FOL}})$ denotes the set of variables of $\mathfrak{L}_{\text{FOL}}$. This allows us to define in the usual way a satisfaction relation $\models_{fs}$ between formulae, assigments and structures, where fs denotes first-order Tarski-style semantics – i.e., the usual semantics for FOL. For example, $\mathfrak{M}, v \models_{fs} R(x_1, ..., x_2)$ iff $\langle v(x_1), ..., v(x_n)\rangle \in R^{\mathfrak{M}}$. We thus say that a stucture $\mathfrak{M}$ *satisfies* a formula ϕ exaclty when there exists an assignment v s.t. $\mathfrak{M}, v \models_{fs} \phi$. Moreover, a formula ϕ (or a set of formulas Γ) is said to be satisfiable if such a structure and assigment exist. Which in its turn serves to define a models relation among sentences (closed formulae) and structures. Assume that ϕ is now a sentence, i.e., a closed formula with no free varìebles. We say that a structure $\mathfrak{M}$ *models* or is a *model* of ϕ, denoted $\mathfrak{M} \models_{fs} \phi$, when, and only when, for any assigment v, $\mathfrak{M}$ satisfies ϕ – a definition that we extend to sets of sentences. (cf. [9, 20]).

Structures can be *contained* in one another – an ordering relation denoted $\preccurlyeq$. This relation is defined as follows:

Definition 3.2.2 *For any two structures* $\mathfrak{M}$ *and* $\mathfrak{M}'$ *over some signature* Σ, $\mathfrak{M}' \preccurlyeq \mathfrak{M}$ *whenever:*

1. $M' \subseteq M$.

2. $R^{\mathfrak{M}'} = R^{\mathfrak{M}} \cap M'^n$, *for every n-ary relation* $R^{\mathfrak{M}}$ *of* $\mathfrak{M}$, *for* $n \geq 1$.

3. $f^{\mathfrak{M}'} = f^{\mathfrak{M}} \upharpoonright M'^n$, *for every n-ary function* $f^{\mathfrak{M}}$ *of* $\mathfrak{M}$, *for* $n \geq 1$.

4. $c^{\mathfrak{M}'} = c^{\mathfrak{M}}$, *for every distinguished element* $c^{\mathfrak{M}}$ *of* $\mathfrak{M}$.

Where $f \upharpoonright S$ denotes the restriction of a function f to $S \subseteq Dom(f)$ (i.e. domain restriction). Moreover, whenever $\mathfrak{M}' \preccurlyeq \mathfrak{M}$ we say that $\mathfrak{M}$ is an *extension* of $\mathfrak{M}'$ and $\mathfrak{M}'$ a *substructure* of $\mathfrak{M}$. We remark further that this order assumes, following its definition, that the structures are of the same *type*, that is, that they have the same number of relations and functions of the same arity and the same number of distinguished elements. We now move to the key notions: that of being a $\forall\exists$ theory (or formula) and that of closure under union of chains.

Definition 3.2.3 *Let ϕ be a FOL sentence. We say that ϕ is a $\forall\exists$ sentence iff ϕ is of the form $\phi = \forall x_1 ... \forall x_n \exists y_1 ... \exists y_m \psi$, that is, a matrix ψ prefixed by n universal quantifiers followed by m existential quantifiers (with $n, m \geq 0$).*

Definition 3.2.4 *Let Γ be a set of FOL sentences. Let $\langle I, <_I \rangle$ be a partial order. We say that Γ is* closed under union of chains *iff for any model $\mathfrak{M}$, and any family $\{\mathfrak{M}_i\}_{i \in I}$ of extensions of $\mathfrak{M}$, s.t. if $i <_I j$, $\mathfrak{M}_i \subseteq \mathfrak{M}_j$, we have that the structure $\mathfrak{M}_\omega$, called the* union structure *and defined below, is also a model of Γ:*

1. *$M_\omega = \bigcup_{t \in T} M_t$.*
2. *Every $R_i^{\mathfrak{M}_\omega}$ is the union of all the relations of the same arity and position among the $\mathfrak{M}_t$'s, for $t \in T$, $i \in I$.*
3. *Every $f_j^{\mathfrak{M}_\omega}$ is the extension of all the functions of the same arity and position among the $\mathfrak{M}_t$'s, for $t \in T$, $j \in J$.*
4. *Every $c_k^{\mathfrak{M}_\omega}$ is a distinguished element among the the $\mathfrak{M}_t$'s, for $t \in T$, $k \in K$.*

The following theorem, whose proof we omit, states an important property of the closure under of chains relationship and about the expressive power of FOL. Indeed, $\forall\exists$ theories have this property, which will be crucial for the next section:

Theorem 3.2.1 (cf. Cori and Lascar, [9]) *Let Γ be a set of FOL sentences. Γ is closed under union of chains iff there exists an $\forall\exists$ set Γ' of FOL sentences equivalent to Γ.*

3.3 Mapping DL-Lite to FOL

As it is has been elsewhere shown (cf.[2]), DL-Lite, like other description logics, can be seen as a fragment of FOL. This will ease the proofs that follow, for we will be moving in well-known ground – i.e. that of FOL and making use of all the properties of its model theory.

Definition 3.3.1 *Let $\mathfrak{L}_{\textit{DL-Lite}}$ denote the signature of DL-Lite and $\mathfrak{L}_{FOL}$ that of FOL. We recursively define the translations $.^{t_x} : \mathfrak{L}_{\textit{DL-Lite}} \rightarrow \mathfrak{L}_{FOL}$ and $.^{t_y} : \mathfrak{L}_{\textit{DL-Lite}} \rightarrow \mathfrak{L}_{FOL}$ as follows:*

1. *For concepts and roles:*
   (a) $A^{t_x} = A[x]$.
   (b) $(\exists R)^{t_x} = \exists y R[x, y]$.
   (c) $(\exists R^-)^{t_x} = \exists x R[x, y]$.
   (d) $(\neg B)^{t_x} = \neg B^{t_x}$.
   (e) $(C \sqcap C')^{t_x} = C^{t_x} \wedge C'^{t_x}$.
   (f) $(\exists R \colon C)^{t_x} = \exists y [R[x, y] \wedge C^{t_y}]$.
   (g) $(\exists R^- \colon C)^{t_x} = \exists x [R[x, y] \wedge C^{t_y}]$.

2. *For A-Box assertions:*

   (a) $(B(c))^{t_x} = B[c]$.

   (b) $(R(c, c'))^{t_x} = R[c, c]$.

3. *For T-Box assertions:*

   (a) $(B \sqsubseteq C)^{t_x} = \forall x[B^{t_x} \to C^{t_x}]$.

   (b) $(R \sqsubseteq R')^{t_x} = \forall x \forall y[R[x, y] \to R'[x, y]]$.

4. *For sets of assertions:* $\Gamma^{t_x} = \{\alpha^{t_x} | \alpha \in \Gamma\}$.

And analogously for $.^{t_y}$. Note that these translations are defined by mutual recursion – remark (1.f) and (1.g) above. As the reader may see, they closely follow the semantics of DL-Lite – they kind of spell it out, so to speak, and generate a fragment of FOL as expressive as DL-Lite. In doing so, we have follwed closely Borgida's translations for very expressive description logics as they appear in [3].

Next, define the mapping $.^t : \mathfrak{L}_{\text{DL-Lite}} \to \mathfrak{L}_{\text{FOL}}$ by putting, for every constant, concept, relation or assertion γ of DL-Lite:

$$\gamma^t = \begin{cases} \gamma^{t_x} & \text{if } \gamma \in Dom(.^{t_x}) \\ \gamma^{t_y} & \text{otherwise.} \end{cases}$$

Now, denote $Im(.^t) \subset \mathfrak{L}_{\text{FOL}}$ the image by $.^t$ in $\mathfrak{L}_{\text{FOL}}$ of DL-Lite in the signature of FOL. Denote FOL^t the fragment of FOL over $Im(.^t)$ thus obtained. FOL^t is a logic of language $\mathfrak{L}_{\text{FOL}^t}$. To prove equivalence of expressive power, we need first some intermediate concepts and properties.

Proposition 3.3.1 *The mapping $.^t$ is an injection from $\mathfrak{L}_{\text{DL-Lite}}$ to $\mathfrak{L}_{\text{FOL}}$ and a bijection from $\mathfrak{L}_{\text{DL-Lite}}$ to $\mathfrak{L}_{\text{FOL}^t}$.*

(*Proof*) Immediate, since, if $C \neq C'$ or $\alpha \neq \alpha'$ for any two pairwise distinct concepts or assertions from $\mathfrak{L}_{\text{DL-Lite}}$, then:

(a) $C^t \neq C'^t$.

(b) $\alpha^t \neq \alpha'^t$.

Now, (a) can be proved by a simple induction on concepts and (b) by case-reasoning over DL-Lite assertions. Bijectivity follows from the fact that $\mathfrak{L}_{\text{FOL}^t}$ coincides by definition with $.^t$'s co-domain. □

Lemma 3.3.1 *A concept C from DL-Lite is satisfiable (w.r.t. description semantics) iff C^t is satisfiable (w.r.t. FOL semantics).*

(*Proof*) It can be easily shown by induction on C that, for some interpretation $\mathfrak{I}$, $d \in \Delta^{\mathfrak{I}}$ and assignement v:

$$d \in C^{\mathfrak{I}} \text{ iff } \mathfrak{I}, v[x/d] \models_{fs} C^t.$$

Which closes the proof. □

Theorem 3.3.1 *DL-Lite* $\sim$ FOL^t.

(*Proof*) (*sketch*) Since the proof is quite lengthy, albeit simple, we will argue informally. Borgida showed that any translation from a description logic to FOL verifying the previous lemma is: (i) A reduction and by the same token an embedding between logics where: (ii) The co-domain constitutes a fragment of FOL with the same expressive power as the initial description logic – provided that the description logic cannot express the transitive closure of relations. Transitive closure is not expressible in FOL (see e.g. [11]). The underlying idea is that $.^t$ is a bijection from $\mathfrak{L}_{\text{DL-Lite}}$ to $\mathfrak{L}_{\text{FOL}^t}$ that preserves satisfaction and that therefore $.^{t^{-1}}$, i.e., $.^t$'s inverse, is defined. Thus (†) holds in both senses: from $\mathfrak{L}_{\text{DL-Lite}}$ to $\mathfrak{L}_{\text{FOL}^t}$ and conversely. And this, as we have seen, implies equivalence in expressive power. □

Chapter 4

DL-Lite and Controlled English

In this section we will prove that the only fragment of English whose meaning representations' expressive power can be compared to that of DL-Lite is COP – the former being at least as expressive as the the meaning represenions of the latter, whereas the converse is false. When we move to COP+TV+DTV, even without taking into consideration distransitive verbs, they are incomparable, even though there are some properties that both DL-Lite and the meaning representations of COP+TV+DTV can express. However negative these results may be, there is all the same something positive about them: the fact that we can easily conjecture that DL-Lite may to a certain extent capture the meaning of more or less if not most of the sentences in COP and COP+TV+DTV. A conjecture that justifies the task we will engage in later, namely that of coining a controlled language, a controlled fragment of English after DL-Lite itself. The general picture is summarized in Figure 4.1.

Theorem 4.0.2 *We have that:*

1. *COP* $\not\lesssim$ *DL-Lite.*
2. *DL-Lite* $\not\lesssim$ *COP.*
3. *COP+TV+DTV* $\not\lesssim$ *DL-Lite.*
4. *DL-Lite* $\not\lesssim$ *COP+TV+DTV.*

(***Proof***) We prove each statement separately. Instead of reasoning over DL-Lite we will reason over the more convenient FOL fragment FOL^t.

(1) Suppose *a contrario* that DL-Lite $\lesssim$ COP. Then every property expressible in COP is expressible also in DL-Lite. Now, consider the sentence:

$$\neg P[c]$$

The models of this sentence are the interpretation structures $\mathfrak{M} = \langle M; P^{\mathfrak{M}}; c^{\mathfrak{M}} \rangle$ where $c^{\mathfrak{M}} \notin M$. But this is impossible, since FOL^t contains no negated atoms.

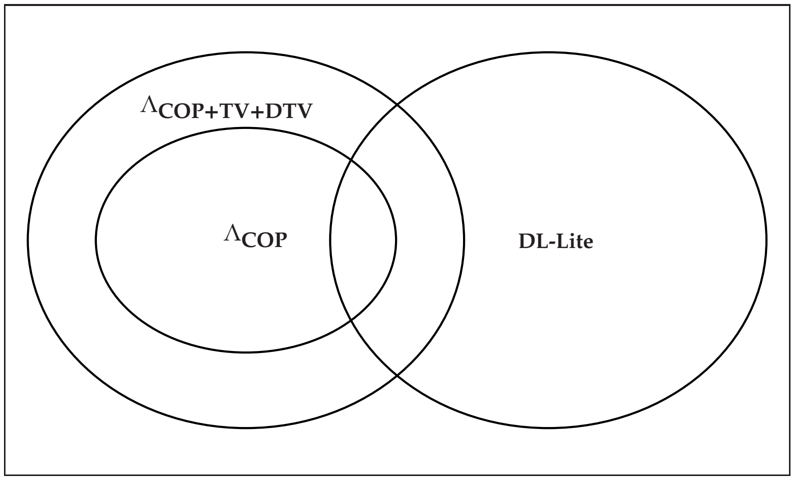

Figure 4.1: *The general picture*

(2) To show this, we will exhibit this time a semantic property that is expressible in FOL(DL-Lite) but not in Λ_{COP}. Consider the sentence:

$$\forall x \forall y[R[x, y] \rightarrow R'[x, y]].$$

Which corresponds to role inclusion assertions in DL-Lite [1]. The models of this sentence are of form $\mathfrak{M} = \langle M; R^{\mathfrak{M}}, R'^{\mathfrak{M}} \rangle$, with $R^{\mathfrak{M}}, R'^{\mathfrak{M}} \subseteq M^2$ and $R^{\mathfrak{M}} \subseteq R'^{\mathfrak{M}}$. But this semantic property cannot be expressed in Λ_{COP} because the signature of this logic contains no relation symbols.

(3) Suppose the contrary. Now, the formulas in FOL^t are all $\exists\forall$ formulas. Therefore, any set of FOL^t sentences will be closed by union of chains. Moreover, by hypothesis, modulo the translation $.^*$, any semantic property that is expressible in COP+TV+DTV is expressible also in FOL^t, in particular:

$$\exists x[P[x] \wedge \forall y[Q[y] \rightarrow R[x, y]]].$$

That is, after prenexing:

$$\exists x \forall y[P[x] \wedge [Q[y] \rightarrow R[x, y]]].$$

Hence the set $\{\exists x \forall y[P[x] \wedge [Q[y] \rightarrow R[x, y]]]\}$ should be closed under union of chains, following the hypothesis. But this is impossible. To show this define a model $\mathfrak{M}$ of this set as follows:

- $M = \mathbb{N}$.
- $P^{\mathfrak{M}} = Q^{\mathfrak{M}} = M$.
- $R^{\mathfrak{M}} = \leq_{\mathbb{N}}$. (i.e. the usual loose order over positive integers).

Define next a sequence $\{\mathfrak{M}_i\}_{i \in \mathbb{N}}$ of extensions of $\mathfrak{M}$ as follows:

[1] At least following our defintion. The reader should bear in mind that there are many different versions of DL-Lite, with slightly different expressivity (cf.[7, 6])

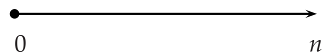

Figure 4.2: *Relation $R^{\mathfrak{M}}$ of model $\mathfrak{M}$.*

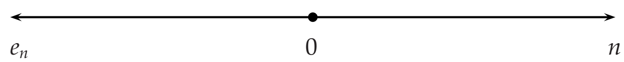

Figure 4.3: *Relation $R^{\mathfrak{M}_\omega}$ of model $\mathfrak{M}_\omega$.*

- $\mathfrak{M}_0$ is defined as follows:
  * $M_0 = M \cup \{e_0\}$.
  * $P^{\mathfrak{M}_0} = Q^{\mathfrak{M}_0} = M_0$.
  * $R^{\mathfrak{M}_0} = R^{\mathfrak{M}} \cup \{\langle e_0, 0\rangle\}$.
- $\mathfrak{M}_{i+1}$ is defined as follows:
  * $M_{i+1} = M_i \cup \{e_{i+1}\}$.
  * $P^{\mathfrak{M}_{i+1}} = Q^{\mathfrak{M}_{i+1}} = M_{i+1}$.
  * $R^{\mathfrak{M}_{i+1}} = R^{\mathfrak{M}_i} \cup \{\langle e_{i+1}, e_i\rangle\}$.

  Now, $\{\mathfrak{M}_i\}_{i\in\mathbb{N}}$ constitutes a chain, since (i) a sequence is a family, (ii) $\langle \mathbb{N}, \leq_{\mathbb{N}}\rangle$ is a partial order and (iii) whenever $i \leq_{\mathbb{N}} j$, $\mathfrak{M}_i \subseteq \mathfrak{M}_j$. Finally, consider the union structure $\mathfrak{M}_\omega$ for this chain. $\mathfrak{M}_\omega$ is not a model of $\exists x \forall y[P[x] \wedge [Q[y] \rightarrow R[x, y]]]$, since the relation $R^{\mathfrak{M}_\omega}$ of $\mathfrak{M}_\omega$ has no least element.

(4) Since COP is but a fragment of COP+TV+DTV:

$$\text{COP} \lesssim \text{COP+TV+DTV}.$$

Trivially holds. This together with (2) entails the result. □

Theorem 4.0.3 *We have that:*

1. *COP overlaps in expressive power with DL-Lite.*
2. *COP+TV+DTV overlaps in expressive power with DL-Lite.*

(*Proof*) We prove only **(1)**, since **(2)** follows inmediately as a trivial corollary. Consider this following typical meaning representation formula for COP:

$$\forall x[P[x] \rightarrow Q[x]].$$

The models of these sentences are the FOL interpretation structures $\mathfrak{M} = \langle M; P^{\mathfrak{M}}, Q^{\mathfrak{M}} \rangle$, where $P^{\mathfrak{M}} \subseteq Q^{\mathfrak{M}}$. But this property can be easily expressed by the DL-Lite T-box assertion:

$$P \sqsubseteq Q.$$

Which closes the proof. □

Remark 4.0.1 Summarizing a bit, we can see that:

- DL-Lite is at least as expressive as COP, which is not a surprise, due to the simplicity of this fragment of English.
- There is no way of comparing the expressive power of DL-Lite w.r.t. satisfiability to that of the tractable (w.r.t. again satisfiability) COP+TV+DTV.
- However, one can see that their expressive power overlaps.
- The idea would thus be that of identifying the greatest (w.r.t. inclusion) fragment of COP and COP+TV+DTV that is contained in this intersection.
- We will do it for COP+TV+DTV. The main idea is to disallow distransitive verbs, since roles are binary relations, plus some more restrictions.

Conclusions

We have studied the expressive power of DL-Lite w.r.t. the meaning representation logics generated or induced by two tractable fragments of English, COP and COP+TV+DTV, and shown that they are not comparable, although they overlap. In doing so, we have introduced a general notion of expressive power for logics defined over a different signature and semantics – namely that of the classes of algrabraic structures they can characterize. Comparison in expressive power among logics is attained by reducing, by means of some translation, the satisfiability or reasoning problem of one logic to that of the other. As a corrollary of our proof, we have established, furthermore, that DL-Lite is closed under the FOL property of union of chains.

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